SELF-CONSISTENT SIMULATIONS AND ANALYSIS OF THE COUPLED-BUNCH INSTABILITY FOR ARBITRARY MULTI-BUNCH CONFIGURATIONS

Gabriele Bassi

Brookhaven National Laboratory, NSLS-II

NOCE 2017

International ICFA mini-Workshop on
NOnlinear dynamics and
Collective Effects in particle beam physics
Arcidosso, Italy / 19 - 22 September 2017

Elettra Sincrotrone Trieste

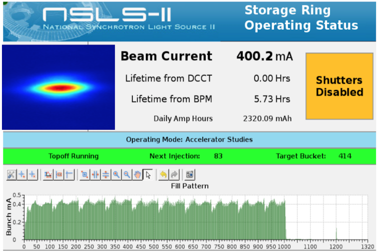
Outline

- NSLS-II Beam Intensity Increasing
- Self-consistent Parallel Tracking Code SPACE
- Coupled Bunch Instability (CBI) for Arbitrary Filling Patterns
  - Theoretical Analysis
  - Benchmarking Theory and Simulations
  - Measurements of the CBI Driven by the Resistive Wall Instability in NSLS-II

NSLS-II Beam Intensity Increasing

02 Jul. 2014	25 mA with CESR-B SC RF cavity
11 Jul. 2014	First time at 50mA
14 Jul. 2014	Shutdown for ID and FE installation
03 Oct. 2104	Start of ID commissioning
23 Oct. 2014	First light on beamline flag!
11 Feb. 2015	**Beamline operations begins at 25 mA**
25 Feb. 2015	50 mA with IVU's magnet gap closed
11 Mar. 2015	First time at 100 mA
15 Apr. 2015	First time at 150 mA
17 Apr. 2015	**Beamline operations begins at 50 mA**
23 Apr. 2015	First time at 200 mA
Jul. 2015	**Beamline operations begins at 150 mA**
28 Jul. 2015	First time at 300 mA
Oct. 2015	**Start operation with Top Off at 150 mA**
04 Jan. 2016	Start operation with 2nd RF cavity
29 Jan. 2016	**Beamline operations begins at 175 mA**

16 Feb. 2016	First time at 350 mA
17 Feb. 2016	**Beamline operations begins at 200 mA**
14 Apr. 2016	**Beamline operations begins at 250 mA**
18 Apr. 2016	First time at 400 mA
16 Feb. 2017	**Beamline operations begins at 275 mA**
05 Apr. 2017	**Beamline operations begins at 300 mA**
20 Jul. 2017	**Beamline operations begins at 325 mA**



Self-consistent Parallel Tracking Code SPACE*

SPACE

(Self-consistent Parallel Algorithm for Collective Effects)

- Efficient study of short and long-range wakefield effects in 6D phase-space via parallel processing communications.
- Study of slow head-tail effect + coupled-bunch instabilities.
- Passive higher harmonic cavity effects with arbitrary fillings.
- Landau damping from betatron tune spread.
- Microwave instability.
- Efficient methods for density estimation from particles.
- Localized wakefield effects.

∗ G. Bassi et al., Phys. Rev. Acc. Beams **19** 024401, 2016.

General Parallel Structure

- M bunches each with N simulations particles distributed to M processors.
- Short-range (single bunch) wakefield interaction calculated in serial (locally).
- Long-range wakefield calculation done in parallel (globally) via master-to-slave processor communications by storing the "history" of moments of the bunches.
- For efficient study of microwave instability, the calculation is done in parallel by distributing N/M simulation particles to M processors.

Analytical Treatment of the CBI for Arbitrary Fillings

- Motivation:
  - Many storage ring modes of operation require multi-bunch configurations that differ from uniform fillings: gap in the fillings for ion clearing (NSLS-II), special multi-bunch configurations with a single high charge bunch (camshaft) .
  - Desire to operate with the most stable multi-bunch configuration.
- The treatment is based on the formulation of an eigenvalue problem defined by the complex frequency shifts of the uniform filling pattern case.
- The numerical solution of the eigenvalue problem allows the study of instability thresholds via the determination of the eigenvalue with the largest imaginary part.
- As a complementary tool to the computation of the eigenvalue spectrum we discuss the Gerschgorin Circle Theorem.
- The application of the Gerschgorin Circle Theorem is useful for
  A) a rapid localization of the eigenvalues in the complex plane.
  B) very efficient perturbative studies of uniform filling patterns.

Transverse Eigenanalysis

The transverse phase space densities $\Psi_m(\tau, \delta, x, p_x, t)$, associated to the $m-$th bunch with N_m particles, satisfy the following system of M-coupled Vlasov equations for $0 \leq m \leq M-1$

$$\begin{aligned}
\frac{\partial \Psi_m}{\partial t} \quad & - \quad \eta\delta \frac{\partial \Psi_m}{\partial \tau} + \frac{\omega_s^2}{\eta}\tau \frac{\partial \Psi_m}{\partial \delta} + p_x \frac{\partial \Psi_m}{\partial x} - \omega_\beta^2 x \frac{\partial \Psi_m}{\partial p_x} \\
& - \quad A_x \sum_{k=0}^{+\infty} \sum_{m'=0}^{M-1} N_{m'} \left[\int_{-\infty}^{\tau} d\tau' W_1\left(\tau - \tau' + a_{m'm}^k T_0\right) \right. \\
& \times \quad \left. \int_{-\infty}^{+\infty} dx' x' \rho_{m'}\left(\tau', x', t - a_{m'm}^k T_0\right) \right] \frac{\partial \Psi_m}{\partial p_x} = 0,
\end{aligned}$$

where $a_{m'm}^k = k + \frac{m'-m}{M}$ and $\rho_m(\tau, x, t) = \int d\delta dp_x \Psi_m(\tau, \delta, x, p_x, t)$.

Equations for the Evolution of Dipole Moments

The time evolution of the dipole moments $\langle x_m \rangle = \int d\tau\, d\delta\, dx\, dp_x\, x\Psi_m$ and $\langle p_{x_m} \rangle = \int d\tau\, d\delta\, dx\, dp_x\, p_x\Psi_m$ is found by integrating by parts the Vlasov equations using the boundary conditions for Ψ_m

$$\frac{d^2}{dt^2}\langle x_m \rangle + \omega_\beta^2 \langle x_m \rangle = -A_x \sum_{k=0}^{+\infty} \sum_{m'=0}^{M-1} f\left(a^k_{m'm} T_0\right) N_{m'} \langle x_{m'} \rangle \left(t - a^k_{m'm} T_0\right),$$

where $f(x) \equiv \int_{-\infty}^{+\infty} d\tau \int_{-\infty}^{+\infty} d\tau' W_1(\tau - \tau' + x)\lambda(\tau)\lambda(\tau')$.

Rearranging and omitting the brackets (x_m understood as $\langle x_m \rangle$)

Time Evolution Bunch Centroid

$$\ddot{x}_m(t) + \omega_\beta^2 x_m(t) = -A_x \sum_{k=0}^{+\infty} f\left(k\frac{T_0}{M}\right) N_{[m+k]} x_{[m+k]} \left(t - k\frac{T_0}{M}\right),$$

where $[m+k] = m + k - M\lfloor (m+k)/M \rfloor$, with $\lfloor x \rfloor$ the floor function (gives the largest integer less or equal to x).

Equations for the Evolution of Mode $\tilde{x}_\mu$

Defining the mode $\tilde{x}_\mu$ by

$$\tilde{x}_\mu(t) = \sum_{m=0}^{M-1} x_m(t) e^{-i2\pi m\mu/M}, \qquad x_m(t) = \frac{1}{M}\sum_{\mu=0}^{M-1} \tilde{x}_\mu(t) e^{i2\pi m\mu/M},$$

it follows that $\tilde{x}_\mu$ are coupled and satisfy the equations of motion

Time Evolution Mode $\tilde{x}_\mu$

$$\ddot{\tilde{x}}_\mu(t) + \omega_\beta^2 \tilde{x}_\mu(t) = -\frac{A_x}{M}\sum_{k=0}^{\infty} f\left(k\frac{T_0}{M}\right) e^{i2\pi\mu k/M} \times \sum_{\mu'=0}^{M-1} \tilde{x}_{\mu'}\left(t - k\frac{T_0}{M}\right) \sum_{m=0}^{M-1} N_m e^{i2\pi m(\mu'-\mu)/M}.$$

Remark: for $N_m = N$ (uniform filling) the modes $\tilde{x}_\mu$ are uncoupled.

Multi-bunch Mode $x_m^{(\mu)}$

The general solution of the non-collective equations of motion $\ddot{\tilde{x}}_\mu(t) + \omega_\beta^2 \tilde{x}_\mu(t) = 0$ reads

$$\begin{aligned}\tilde{x}_\mu(t) &= A_1 e^{i\omega_\beta t} + A_2 e^{-i\omega_\beta t} = \frac{1}{2}\Big[\Big(\tilde{x}_\mu(0) - \frac{i}{\omega_\beta}\dot{\tilde{x}}_\mu(0)\Big) e^{i\omega_\beta t} \\ &+ \Big(\tilde{x}_\mu(0) + \frac{i}{\omega_\beta}\dot{\tilde{x}}_\mu(0)\Big) e^{-i\omega_\beta t}\Big].\end{aligned}$$

$x_m \in \mathbb{R} \Longrightarrow \tilde{x}_\mu = \tilde{x}^\star_{M-\mu}$, thus define the multi-bunch mode as

$$\begin{aligned}x_m^{(\mu)}(t) &= \frac{1}{M}\Big(\tilde{x}_\mu(t) e^{i2\pi m\mu/M} + \tilde{x}_{M-\mu}(t) e^{-i2\pi m\mu/M}\Big) \\ &= \frac{1}{M}\Big(\tilde{x}_\mu(t) e^{i2\pi m\mu/M} + \tilde{x}^\star_\mu(t) e^{-i2\pi m\mu/M}\Big) \\ &= \frac{2}{M}\Big(\mathrm{Re}\tilde{x}_\mu(t)\cos\frac{2\pi\mu m}{M} - \mathrm{Im}\tilde{x}_\mu(t)\sin\frac{2\pi\mu m}{M}\Big).\end{aligned}$$

Perturbative Solution

Look for a perturbative solution identifying the perturbation by the parameter ϵ.
Without loss of generality, assume for the perturbative solution the form

Perturbative Solution Mode $\tilde{x}_\mu$

$$\tilde{x}_\mu(t) = A_1 e^{i(\omega_\beta+\epsilon\Omega)t} + A_2 e^{-i(\omega_\beta+\epsilon\Omega)t} = a_\mu e^{-i(\omega_\beta+\epsilon\Omega)t}, \qquad \Omega \in \mathbb{C},$$

where $a_\mu = \tilde{x}_\mu(0)$ and we chose $A_1 = 0$.
Defining $\tau^{-1} \equiv \mathrm{Im}\Omega$, $\omega_r \equiv \mathrm{Re}\Omega$ and assuming $\mathrm{Im}\tilde{x}_\mu(0) = 0$, the multi-bunch mode $x_m^{(\mu)}$ takes the form

Perturbative Solution Multi-bunch Mode $x_m^{(\mu)}$

$$x_m^{(\mu)}(t) = a e^{\frac{t}{\tau}} \left(\cos\frac{2\pi\mu m}{M} \cos(\omega_\beta+\omega_r)t + \sin\frac{2\pi\mu m}{M} \sin(\omega_\beta+\omega_r)t \right),$$

where $a = 2\mathrm{Re}\tilde{x}_\mu(0)/M$ and we set the perturbation parameter $\epsilon = 1$.

Eigenvalue Equation

The perturbative solution for $\tilde{x}_\mu$ leads to the eigenvalue equation

$$\left[\frac{A_x N}{2\omega_\beta}\sum_{k=0}^{\infty} f\left(k\frac{T_0}{M}\right) e^{i2\pi\mu k/M} e^{i\omega_\beta k T_0/M} - \Omega\right] a_\mu$$

$$+\sum_{\substack{\mu'=0\\ \mu'\neq\mu}}^{M-1}\left[\frac{A_x}{2\omega_\beta M}\sum_{k=0}^{\infty} f\left(k\frac{T_0}{M}\right) e^{i2\pi\mu k/M} e^{i\omega_\beta k T_0/M}\sum_{m=0}^{M-1} N_m e^{i2\pi m(\mu'-\mu)/M}\right] a_{\mu'} = 0.$$

or, equivalently

Eigenvalue Equation for Arbitrary Filling Patterns

$$(\mathbf{B}-\Omega\mathbf{I})\mathbf{a} = 0,\quad B_{\mu\mu'} = \frac{\Omega_\mu^U}{NM}\sum_{m=0}^{M-1} N_m e^{i2\pi m(\mu'-\mu)/M},\quad \mathbf{a} = [a_0, .., a_{M-1}]^T$$

where Ω_μ^U are the eigenvalues of the uniform filling pattern case ($B_{\mu\mu'} = \Omega_\mu^U$ if $\mu' = \mu$, 0 otherwise)

$$\Omega_\mu^U = -i\frac{I_b M c}{4\pi(E_0/e)\nu_\beta}\sum_{p=-\infty}^{+\infty}\left|\tilde{\lambda}\left[(pM+\mu)\omega_0+\omega_\beta\right]\right|^2 Z_1^\perp\left[(pM+\mu)\omega_0+\omega_\beta\right].$$

Solving for the characteristic polynomial $p(\Omega) = |\mathbf{B} - \Omega\mathbf{I}| = 0$ and assuming M distinct eigenvalues Ω_m, the general solution $\tilde{x}^g_\mu(t)$ is given by

$$\tilde{x}^g_\mu(t) = \sum_{m=0}^{M-1} c_m a_{\mu m} e^{-i(\omega_\beta + \Omega_m)t},$$

where $\mathbf{a}_m = [a_{0\,m}, .., a_{M-1\,m}]^T$ are the eigenvectors associated to the eigenvalues Ω_m.

Since the sum of the eigenvalues of $\mathbf{B}$ is equal to the trace of $\mathbf{B}$, it follows that the sum of the complex frequency shifts Ω_μ for arbitrary filling patterns is equal to the sum of the complex frequency shifts Ω^U_μ for uniform filling patterns

$$\sum_{\mu=0}^{M-1} \Omega_\mu = \mathrm{Tr}\,\mathbf{B} \quad \Longrightarrow \quad \sum_{\mu=0}^{M-1} \Omega_\mu = \sum_{\mu=0}^{M-1} \Omega^U_\mu .$$

Gerschgorin Circle Theorem*

Estimation of eigenvalue spectrum very accurate for strictly diagonally dominant matrices (off-diagonal terms small with respect to diagonal terms), i.e. for a $n \times n$ complex matrix $\mathbf{A} = (a_{ij})$ for which

$$|a_{ii}| > \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}|, \qquad 0 \leq i \leq n.$$

Gerschgorin's Theorem

If λ is an eigenvalue of an $n \times n$ matrix $\mathbf{A} = (a_{ij})$, then, for some i,

$$|\lambda - a_{ii}| \leq \sum_{\substack{j=1 \\ j \neq i}}^{n} |a_{ij}| \equiv r_i, \qquad 0 \leq i \leq n.$$

r_i defines the radius of a Gerschgorin row disc. Defining similarly the radius of a Gerschgorin column disc, we have that all of the eigenvalues of the matrix $\mathbf{A}$ are contained in the intersection of the union of all the row discs and column discs.

*S. Gerschgorin, Izv. Akad. Nauk. USSR Otd. Fiz-Mat. Nauk 7, 749-754, (1931).

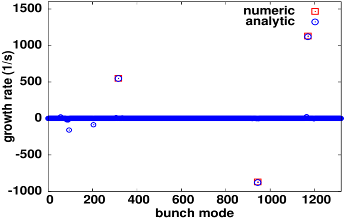
Benchmarking Theory and Simulations

Study of the coupled-bunch instability driven by the HOMs of the 7-cell PETRA-III RF Cavity (used during commissioning phase 1 of the NSLS-II storage ring).

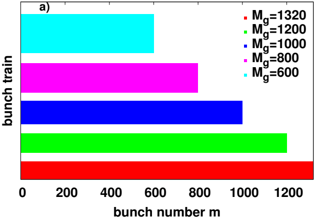
TABLE : Transverse HOMs of the 7-cell PETRA-III RF Cavity

f_r, MHz	$R_{sh,\perp}$, MΩ/m	$Q_\perp$	$R_{sh,\perp}/Q_\perp$, Ω/m
860.25	14.7	55700	263.91
867.12	17.5	56800	308.1
869.55	56.1	58200	963.92
870.96	19.7	59400	331.65
1043.53	83.6	40400	2069.31
1047.44	26.2	40900	640.59
1089.13	17.0	49400	344.13
1465.13	15.5	54600	283.88
1545.34	26.8	44300	604.97

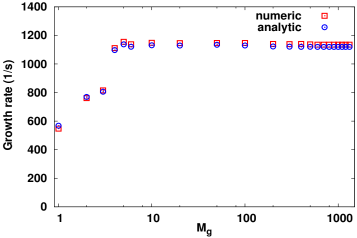
Uniform Bunch Train with a Gap



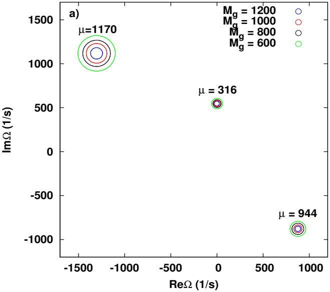
Growth rates uniform filling



Bunch trains with different gaps



Fastest instability vs. bunch train with a gap



Fast Eigenvalue Estimation: Gerschgorin Circle Theorem

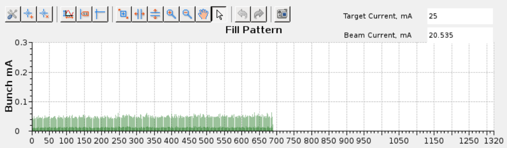
NSLS-II Measurements with Gap in the Fill Pattern

(Operational lattice with 3DW gaps closed at zero linear chromaticity)

Calculated CBI threshold from resistive wall impedance $I_{th} = 22\text{mA}$.

Fill Pattern Monitor

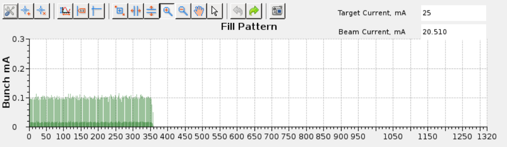
I_b (single bunch current)



Uniform I_b

$M = 688$ (gap = 632)

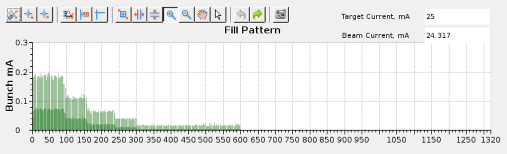
$I_{th} = 20.5\text{mA}$



Uniform I_b

$M = 352$ (gap = 968)

$I_{th} = 20.5\text{mA}$



Non-uniform I_b

$M = 600$ (gap = 620)

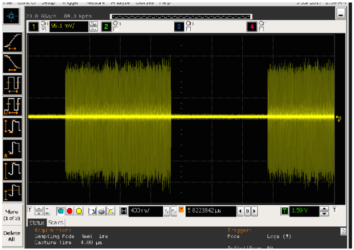
$I_{th} = 24.3\text{mA}$

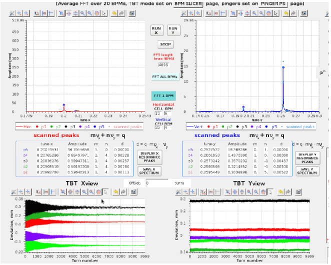
NSLS-II Measurements with Gap in the Fill Pattern

Fill Pattern from Oscilloscope

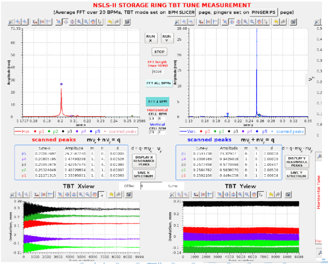
Tune Measurement from TBT Data

Uniform I_b
$M = 688$
$I_{th} = 20.5\text{mA}$





Uniform I_b
$M = 1284$
$I_{th} = 20.5\text{mA}$

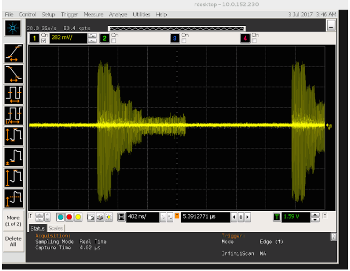


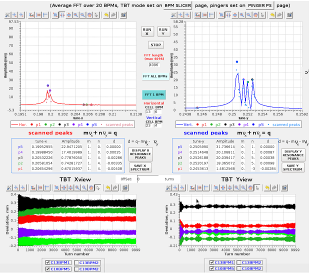
NSLS-II Measurements with Gap in the Fill Pattern

Fill Pattern from Oscilloscope

Tune Measurement from TBT Data

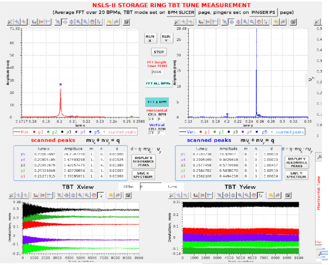
Non-uniform I_b
$M = 600$
$I_{th} = 24.3\text{mA}$





$I_{th} = 24.3\text{mA}$ (higher threshold possibly explained by betatron tune spread across the bunch train induced by single bunch effects)

Uniform I_b
$M = 352$
$I_{th} = 20.5\text{mA}$



THANK YOU!

BACK-UP SLIDES

Elementary case: two bunches

For $M = 2$, and with $2N = N_T$, the eigenvalue problem reads

$$\begin{vmatrix} \Omega_0^U - \Omega & N_-\Omega_0^U \\ N_-\Omega_1^U & \Omega_1^U - \Omega \end{vmatrix} = 0, \quad N_- = \frac{N_0 - N_1}{N_0 + N_1}, \quad N_0 + N_1 = 2N,$$

and is easily solved with eigenvalues

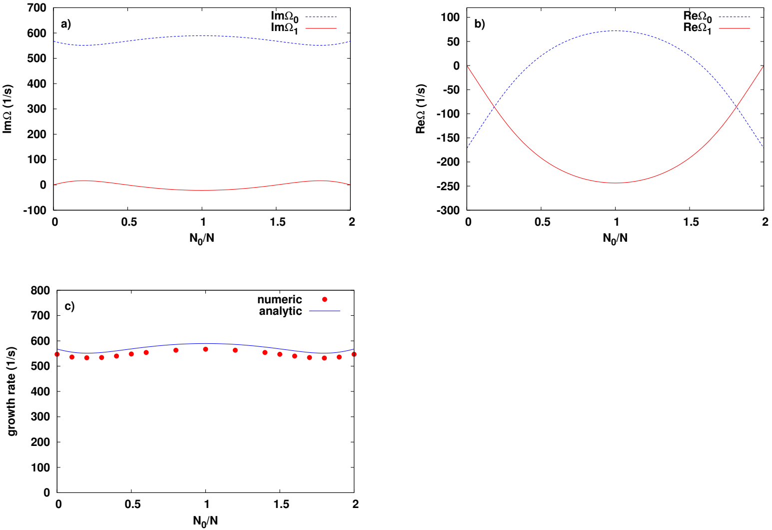
$$\Omega_{0,1} = \frac{\Omega_0^U + \Omega_1^U}{2} \pm \frac{1}{2}\sqrt{(\Omega_0^U - \Omega_1^U)^2 + 4N_-^2\Omega_0^U\Omega_1^U}, \quad \Omega_0 + \Omega_1 = \Omega_0^U + \Omega_1^U,$$

and corresponding eigenvectors

$$\mathbf{a}_0 = \left[1, -\frac{2\Omega_1^U N_-}{\Omega_1^U - \Omega_0^U - \sqrt{(\Omega_0^U - \Omega_1^U)^2 + 4N_-^2\Omega_0^U\Omega_1^U}}\right]^T$$

$$\mathbf{a}_1 = \left[\frac{2\Omega_0^U N_-}{\Omega_1^U - \Omega_0^U - \sqrt{(\Omega_0^U - \Omega_1^U)^2 + 4N_-^2\Omega_0^U\Omega_1^U}}, 1\right]^T.$$

Elementary case: two bunches



Elementary Case 2: Three Bunches

In the case $M = 3$ we discuss the configuration $N_0 = N_1 = 3N/2, N_2 = 0$, with $3N = N_T$, which describes a configuration with a missing bunch. The corresponding eigenvalue problem reads

$$|\mathbf{B} - \Omega \mathbf{I}| = \begin{vmatrix} \Omega_0^U - \Omega & a\Omega_0^U & a^*\Omega_0^U \\ a^*\Omega_1^U & \Omega_1^U - \Omega & a\Omega_1^U \\ a\Omega_2^U & a^*\Omega_2^U & \Omega_2^U - \Omega \end{vmatrix} = 0, \quad a = \frac{1+\sqrt{3}i}{2},$$

where * denotes complex conjugate. It follows that

$$\mathbf{B} - \Omega \mathbf{I}| = -\Omega \left[\Omega^2 - \mathrm{Tr}\, \mathbf{B}\Omega + \frac{3}{4} \left(\Omega_0^U \Omega_1^U + \Omega_0^U \Omega_2^U + \Omega_1^U \Omega_2^U \right) \right] = 0,$$

where $\mathrm{Tr}\, \mathbf{B} = \Omega_0^U + \Omega_1^U + \Omega_2^U$.